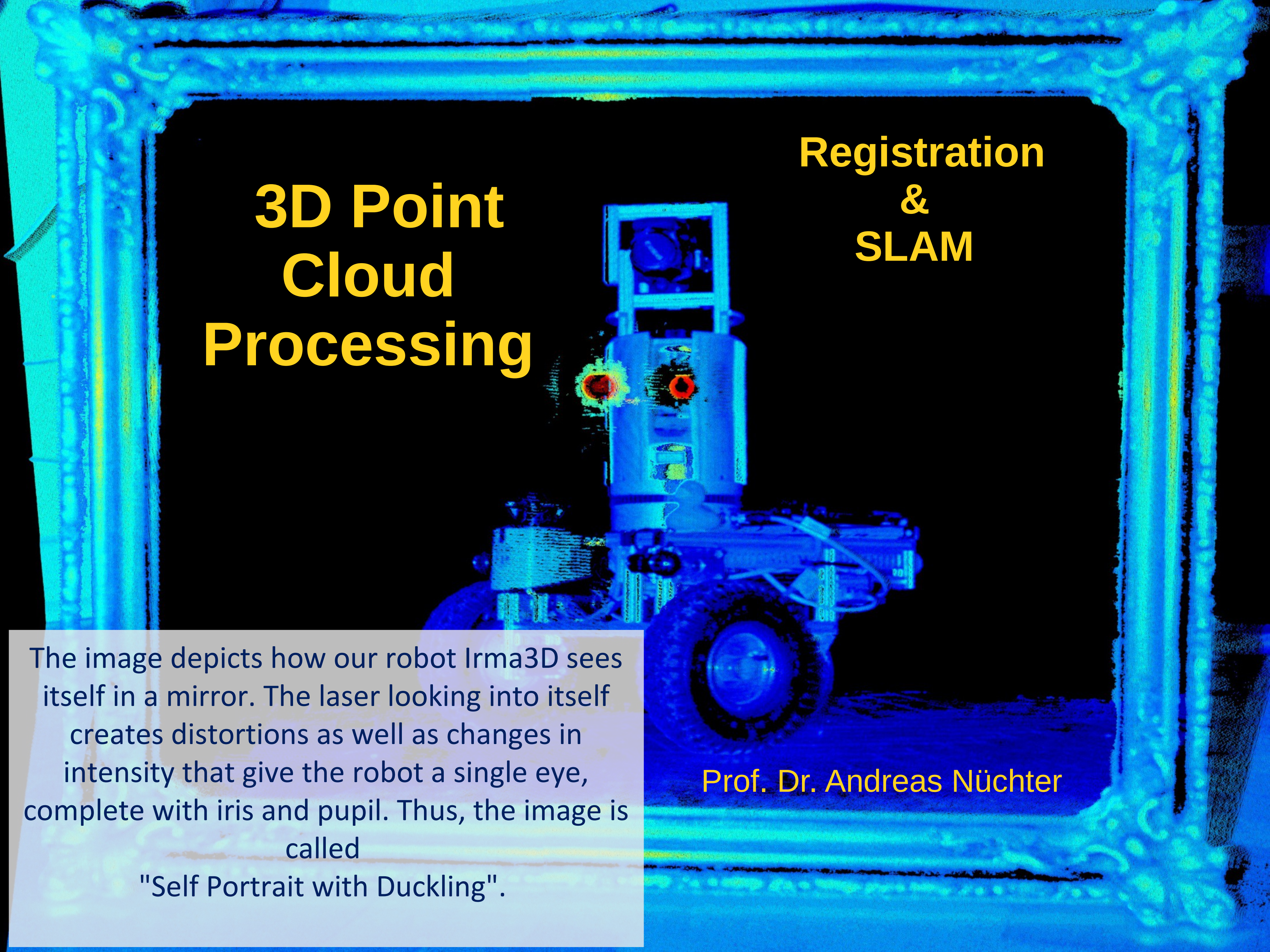


3D Point Cloud Processing

Registration & SLAM

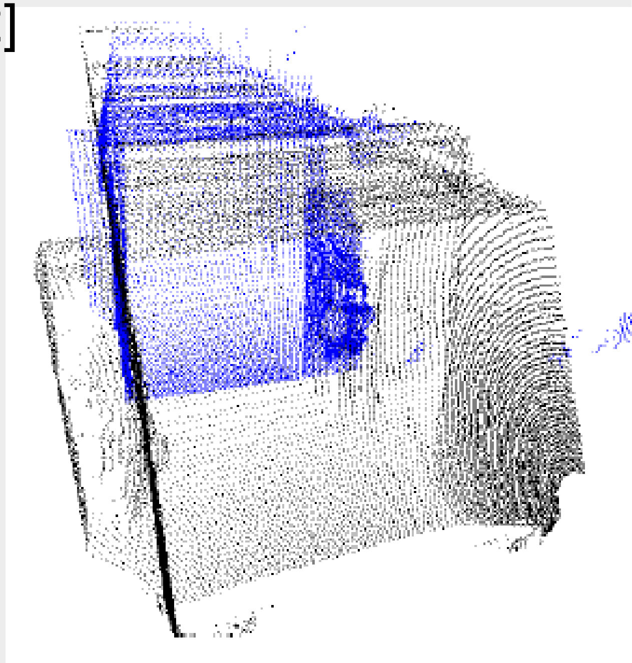
The image depicts how our robot Irma3D sees itself in a mirror. The laser looking into itself creates distortions as well as changes in intensity that give the robot a single eye, complete with iris and pupil. Thus, the image is called
"Self Portrait with Duckling".

Prof. Dr. Andreas Nüchter

The ICP Algorithm (1)

Scan registration Put two independent scans into one frame of reference

Iterative Closest Point algorithm [Besl/McKay 1992]



For prior point set M ("model set") and data set D

1. Select point correspondences $w_{i,j}$ in {0,1}
2. Minimize for rotation $\mathbf{R}$, translation $\mathbf{t}$

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N_m}\sum_{j=1}^{N_d} w_{i,j} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})\right|\right|^2$$

3. Iterate **1.** and **2.**

SVD-based calculation of rotation

- works in 3 translation plus 3 rotation dimensions
  $\Rightarrow$ 6D SLAM with closed loop detection and global relaxation.

The ICP Algorithm (2)

Closed form (one-step) solution for minimizing of the error function

1. **Cancel the double sum:**

$$
\begin{aligned}
E(\mathbf{R},\mathbf{t}) &= \sum_{i=1}^{N_m}\sum_{j=1}^{N_d} w_{i,j} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})\right|\right|^2 \\
&\propto \frac{1}{N}\sum_{i=1}^{N} \left|\left|\mathbf{m}_i - (\mathbf{R}\mathbf{d}_i + \mathbf{t})\right|\right|^2,
\end{aligned}
$$

2. **Compute centroids of the matching points**

$$
\mathbf{c}_m = \frac{1}{N}\sum_{i=1}^{N} \mathbf{m}_i, \qquad \mathbf{c}_d = \frac{1}{N}\sum_{i=1}^{N} \mathbf{d}_j
$$

$$
M' = \{\mathbf{m}'_i = \mathbf{m}_i - \mathbf{c}_m\}_{1,\ldots,N}, \qquad D' = \{\mathbf{d}'_i = \mathbf{d}_i - \mathbf{c}_d\}_{1,\ldots,N}.
$$

3. **Rewrite the error function**

$$
E(\mathbf{R},\mathbf{t}) = \frac{1}{N}\sum_{i=1}^{N} \left|\left|\mathbf{m}'_i - \mathbf{R}\mathbf{d}'_i - \underbrace{(\mathbf{t} - \mathbf{c}_m + \mathbf{R}\mathbf{c}_d)}_{=\tilde{\mathbf{t}}}\right|\right|^2
$$

The ICP Algorithm (3)

Closed form (one-step) solution for minimizing of the error function

3. **Rewrite the error function**

$$
\begin{aligned}
E(\mathrm{R}, \mathrm{t}) &= \frac{1}{N} \sum_{i=1}^{N} ||\mathrm{m}'_i - \mathrm{R}\mathrm{d}'_i - \underbrace{(\mathrm{t} - \mathrm{c}_m + \mathrm{R}\mathrm{c}_d)}_{=\tilde{\mathrm{t}}}||^2 \\
&= \frac{1}{N} \sum_{i=1}^{N} \left|\left|\mathrm{m}'_i - \mathrm{R}\mathrm{d}'_i\right|\right|^2 - \frac{2}{N} \tilde{\mathrm{t}} \cdot \sum_{i=1}^{N} \left(\mathrm{m}'_i - \mathrm{R}\mathrm{d}'_i\right) + \frac{1}{N} \sum_{i=1}^{N} \left|\left|\tilde{\mathrm{t}}\right|\right|^2 .
\end{aligned}
$$

- **Minimize only the first term! (The second is zero and the third has a minimum for $\tilde{\mathrm{t}} = 0$).**

$$
E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}'_i - \mathrm{R}\mathrm{d}'_i\right|\right|^2 .
$$

Arun, Huang und Blostein suggest a solution based on the singular value decomosition.

K. S. Arun, T. S. Huang, and S. D. Blostein. Least square fitting of two 3-d point sets. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 9(5):698 – 700, 1987.

The ICP Algorithm (4)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m_{ix}' d_{ix}',\ S_{xy} = \sum_{i=1}^{N} m_{ix}' d_{iy}',\ \ldots$ **,then the optimal solution for** $E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2$ **is** $\mathrm{R} = \mathrm{V}\mathrm{U}^T$ **with** $\mathbf{H} = \mathrm{U}\Lambda\mathrm{V}^T$ **from the SVD.**

Proof:

$$E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2 .$$

Rewrite

$$E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i'\right|\right|^2 - 2\sum_{i=1}^{N} \mathrm{m}_i' \cdot \mathrm{R}\mathrm{d}_i' + \sum_{i=1}^{N} \left|\left|\mathrm{d}_i'\right|\right|^2 .$$

Rotation is length preserving, i.e., maximize the term

$$\sum_{i=1}^{N} \mathrm{m}_i' \cdot \mathrm{R}\mathrm{d}_i' = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{R}\mathrm{d}_i'$$

The ICP Algorithm (5)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m'_{ix} d'_{ix}, \; S_{xy} = \sum_{i=1}^{N} m'_{ix} d'_{iy}, \; \ldots$ **,then the optimal solution for** $E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2$ **is** $\mathrm{R} = \mathbf{V}\mathbf{U}^T$ **with** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$ **from the SVD.**

Proof: $$\sum_{i=1}^{N} \mathrm{m}_i' \cdot \mathrm{R}\mathrm{d}_i' = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{R}\mathrm{d}_i'$$

Rewrite using the trace of a matrix

$$\mathrm{Trace}\left(\sum_{i=1}^{N} \mathrm{R}\mathrm{d}_i' \mathrm{m}_i'^T\right) = \mathrm{Trace}\left(\mathbf{R}\mathbf{H}\right)$$

Lemma: For all positiv definite matrices $\mathbf{A}\mathbf{A}^T$ and all orthonormal matrices B the following equation holds: $\mathrm{Trace}\left(\mathbf{A}\mathbf{A}^T\right) \geq \mathrm{Trace}\left(\mathbf{B}\mathbf{A}\mathbf{A}^T\right)$

$\square$

The ICP Algorithm (6)

Theorem: Given a 3 x 3 correlation matrix

$$\mathbf{H} = \sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{d}_i' = \begin{pmatrix} S_{xx} & S_{xy} & S_{xz} \\ S_{yx} & S_{yy} & S_{yz} \\ S_{zx} & S_{zy} & S_{zz} \end{pmatrix}$$

with $S_{xx} = \sum_{i=1}^{N} m_{ix}' d_{ix}',\ S_{xy} = \sum_{i=1}^{N} m_{ix}' d_{iy}',\ \ldots$ **,then the optimal solution for** $E(\mathrm{R}, \mathrm{t}) = \sum_{i=1}^{N} \left|\left|\mathrm{m}_i' - \mathrm{R}\mathrm{d}_i'\right|\right|^2$ **is** $\mathrm{R} = \mathbf{V}\mathbf{U}^T$ **with** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$ **from the SVD.**

Proof: Suppose the singular value decomposition of $\mathbf{H}$ **is** $\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T$

$\mathbf{U}$ **and** $\mathbf{V}$ **are orthonormal 3 x 3 and** Λ **a diagonal matrix without negative entries .**

$$\mathrm{R} = \mathbf{V}\mathbf{U}^T.$$

R **is orthonormal and**

$$\begin{aligned} \mathrm{R}\mathbf{H} &= \mathbf{V}\mathbf{U}^T\mathbf{U}\Lambda\mathbf{V}^T \\ &= \mathbf{V}\Lambda\mathbf{V}^T \end{aligned}$$

And using the lemma it is $\mathrm{Trace}\left(\mathrm{R}\mathbf{H}\right) \geq \mathrm{Trace}\left(\mathbf{B}\mathrm{R}\mathbf{H}\right).$

Therefore R **maximizes**

$$\sum_{i=1}^{N} \mathrm{m}_i'^T \mathrm{R}\mathrm{d}_i'$$

$\square$

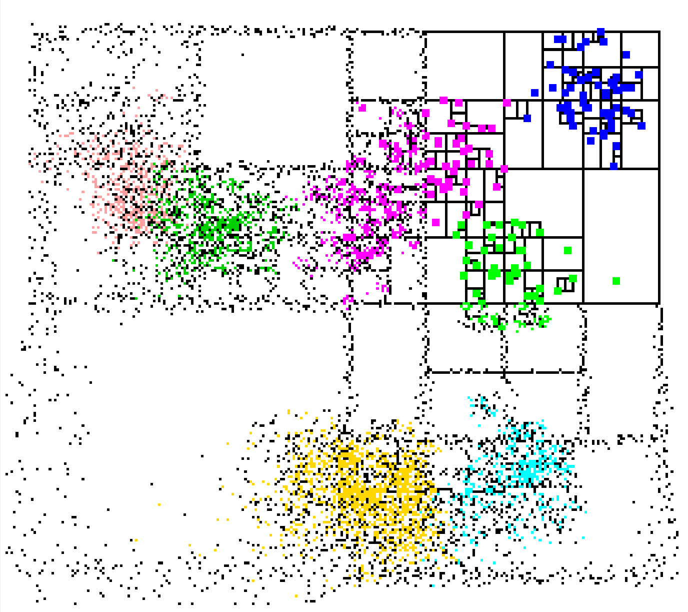
The ICP Algorithm (7)

- Estimating the transformation can be accomplished very fast O(n)

- Closest point search
  - Naïve O(n^2), i.e., brute force
  - K-d trees for searching in logarithmic time

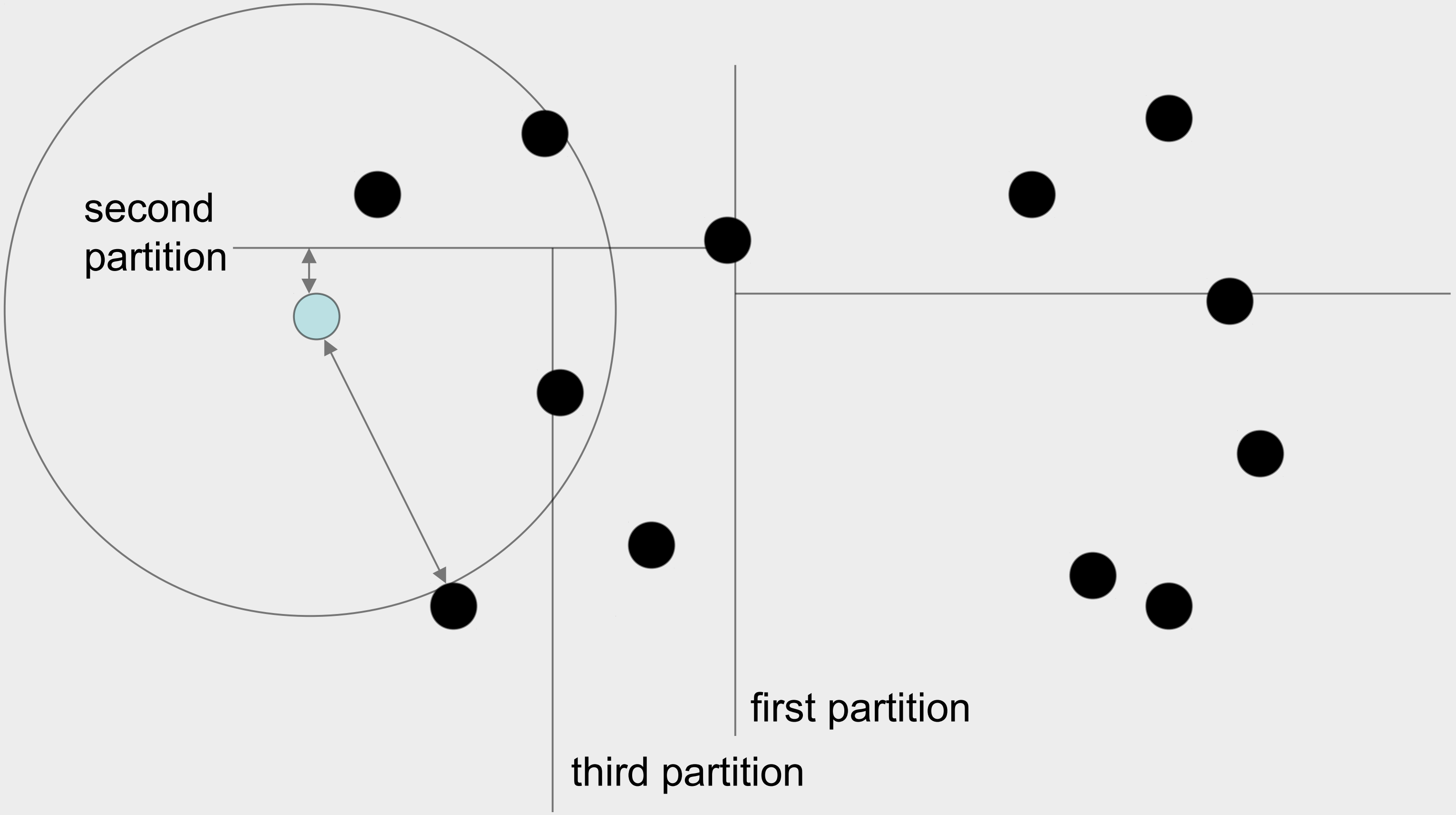
  Recommendation: Start with

  ANN: A Library for Approximate Nearest Neighbor Searching by David M. Mount and Sunil Arya (University of Maryland)
    - Easy to use
    - Many different methods are available
    - Quite fast



`http://www.cs.umd.edu/~mount/ANN/`

K-d Tree based NNS (1)

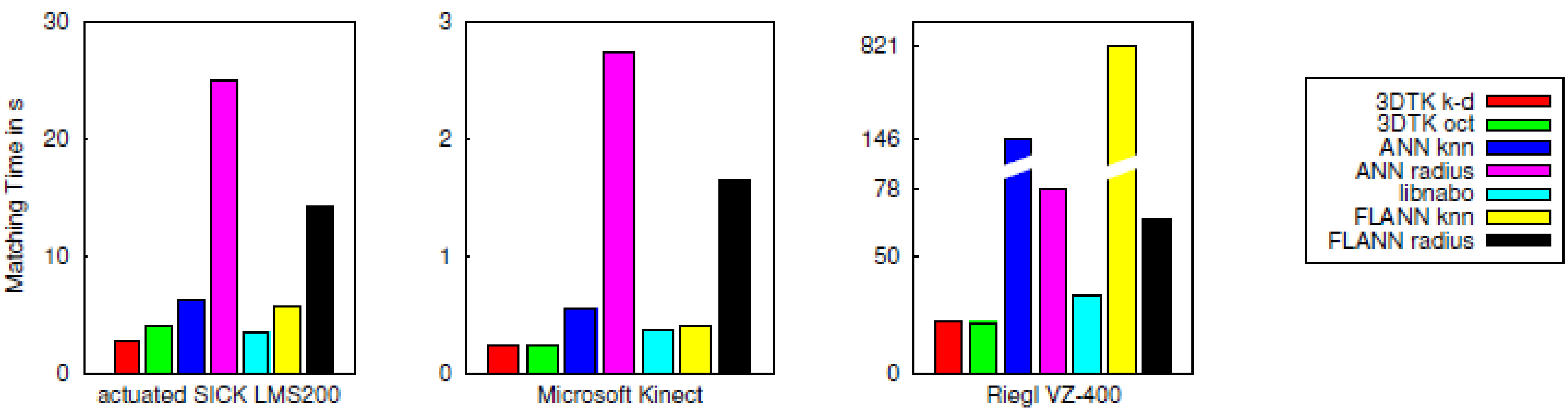


- One has to search all buckets according to the ball-within-bounds-test. ⇨ Backtracking

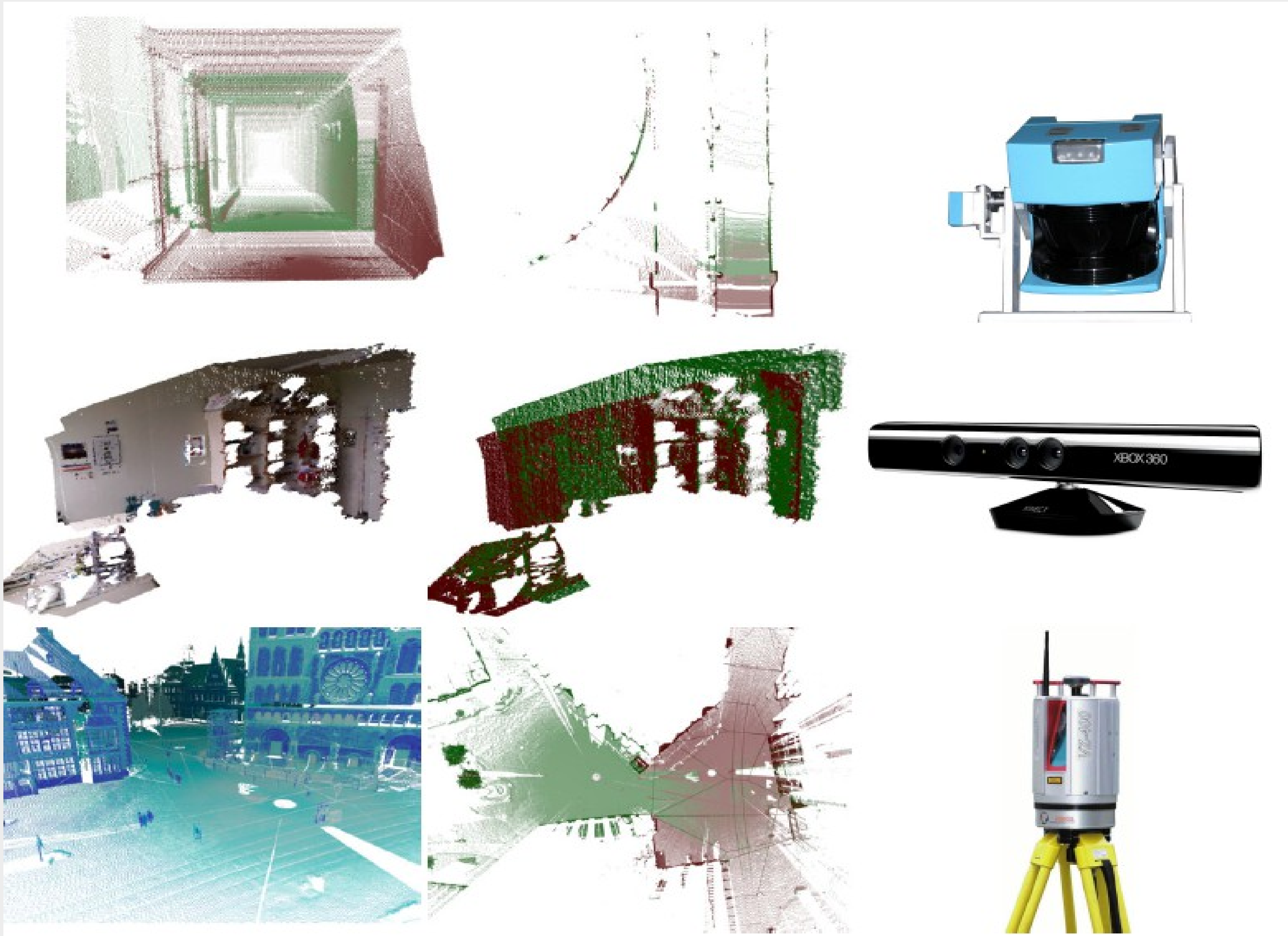
NNS Search – the Critical Issue

Properties for all tested NNS libraries.

Library	revision	Data structure	k-NN search	fixed radius	ranged search	optimized for
3DTK [2]	rev. 470	k-d tree	×	×	✓	shape registration
3DTK	rev. 470	octree	×	×	✓	shape registration & efficient storage
ANN [3]	Ver. 1.1.1	k-d tree	✓	✓	×	
CGAL [4]	Ver. 3.5.1-1	k-d tree	×	✓	×	
FLANN [5]	bcf3a56e5fed2d4dc3a340725fa341fa36ef79a4	k-d tree	✓	✓	×	high dimensions
libnabo [6]	Ver. 1.0.0	k-d tree	✓	×	✓	
SpatialIndex [7]	Ver. 1.4.0-1.1	R-tree	✓	×	×	
STANN [8]	Ver. 0.71 beta	SFC	✓	×	×	multithreading

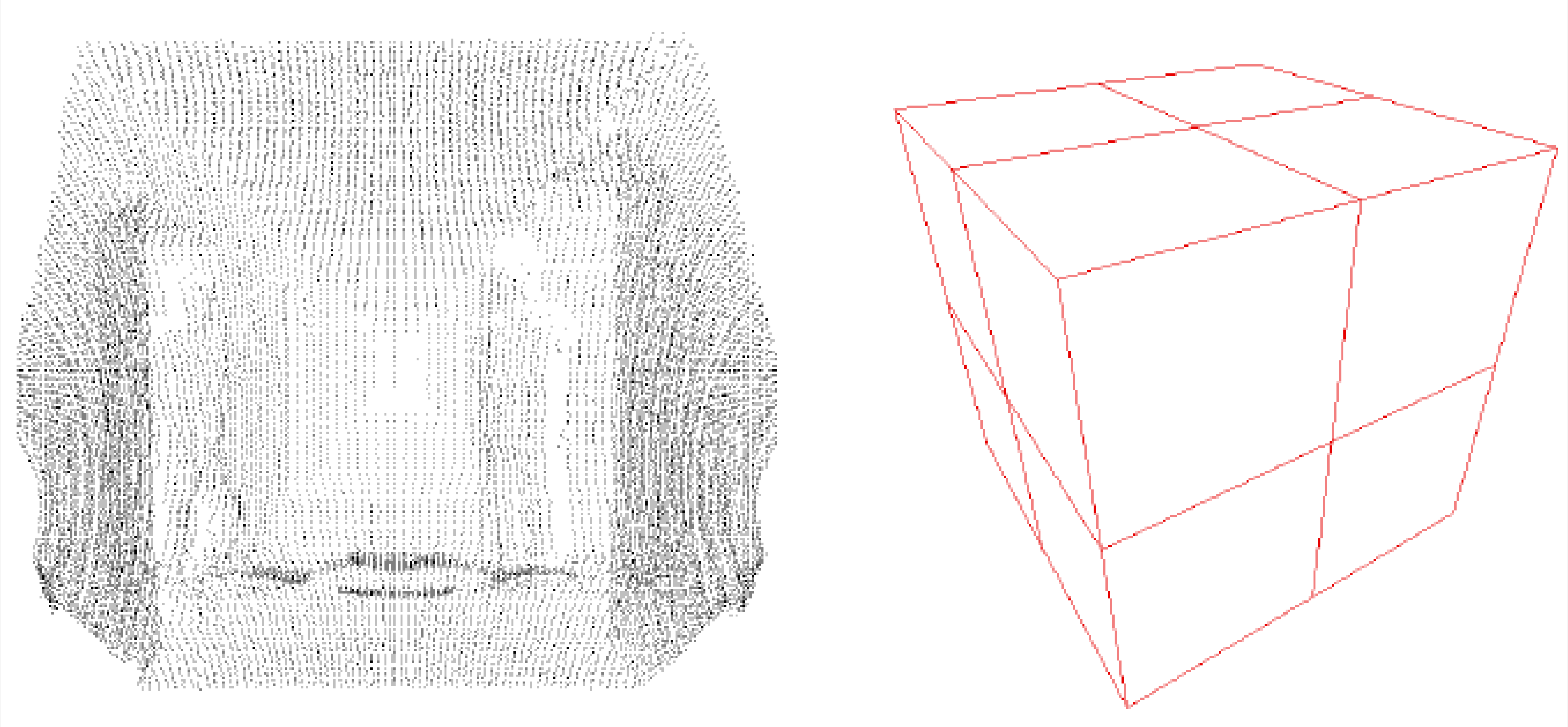


NNS Search – the Critical Issue



The ICP Algorithm (8)

- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud



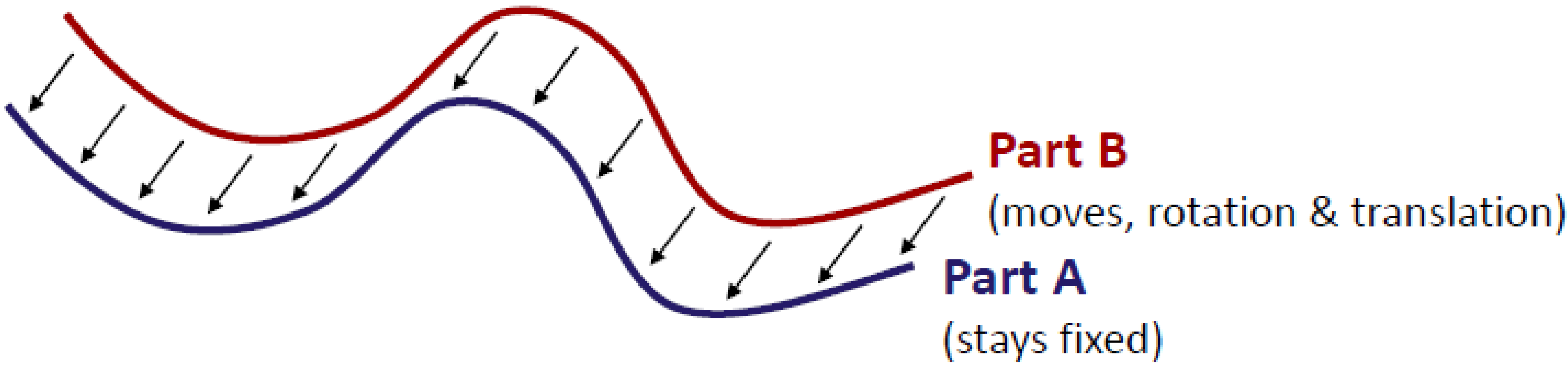
The ICP Algorithm (9)

- Point reduction – another key for fast ICP algorithms
  - Start with cube surrounding the 3D point cloud
  - Divide

- Another key issue: maximal point-to-point distance.

Registering Surfaces (1)

- Given



The main idea:

- Pairwise matching technique
- We want to minimize the distance between the two parts
- We set up a variational problem
- Minimize distance “energy” by rigid motion of one part

Registering Surfaces (2)

Problem:

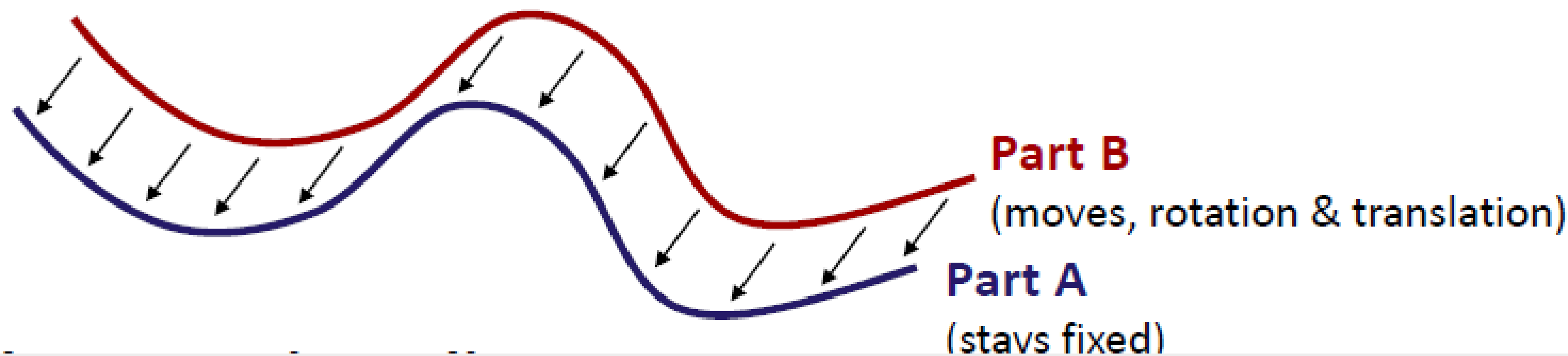
- How to compute the distance
- This is simple if we know the corresponding points.
- Of course, we have in general no idea of what corresponds...

- ICP-idea: set closest point as corresponding point

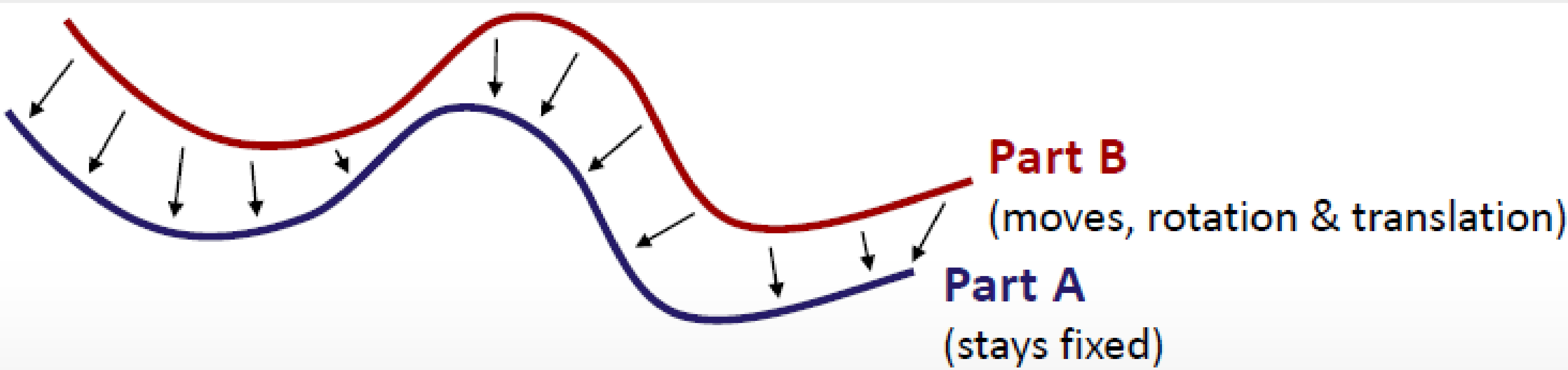
- Full algorithm:
  - Compute closest point points
  - Minimize distance to these closest points by a rigid motion
  - Recompute new closest points and iterate

Registering Surfaces (3)

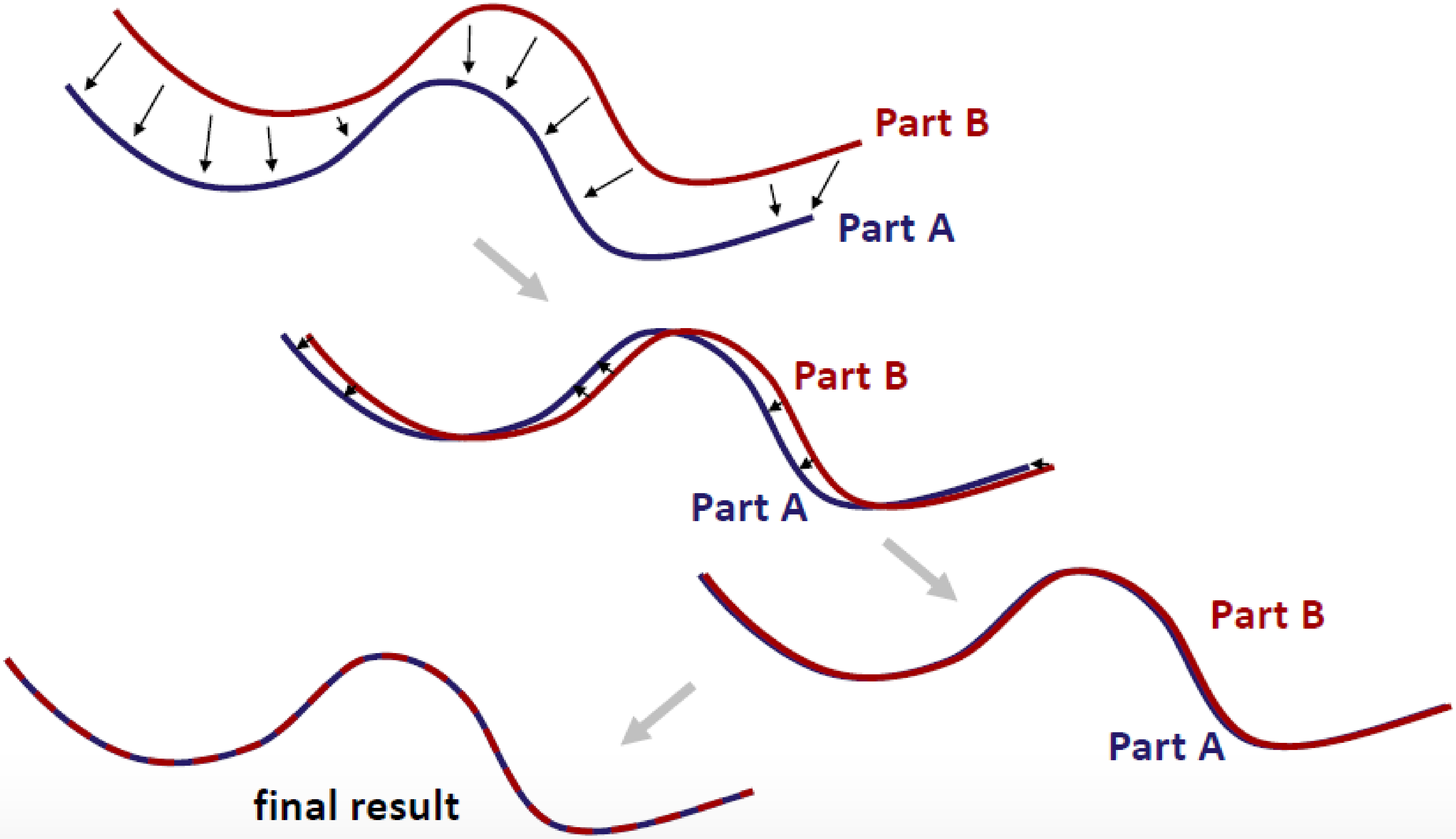
- Distances



- Closest Point Distances

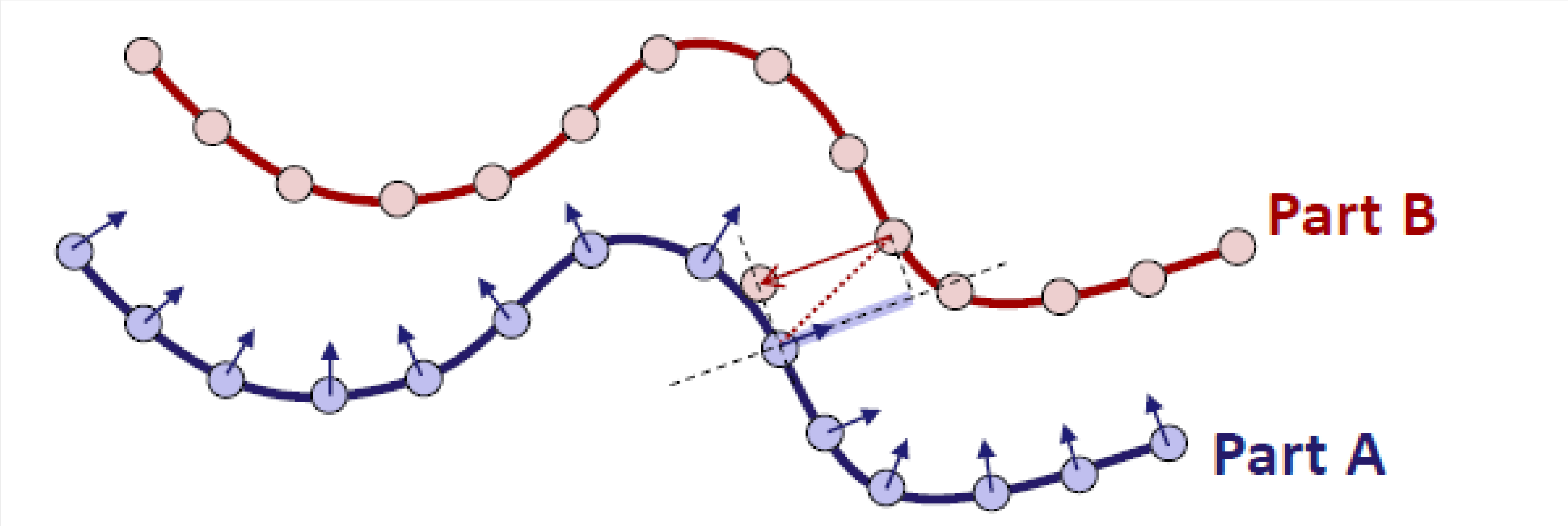


Registering Surfaces (4) – ICP iterations

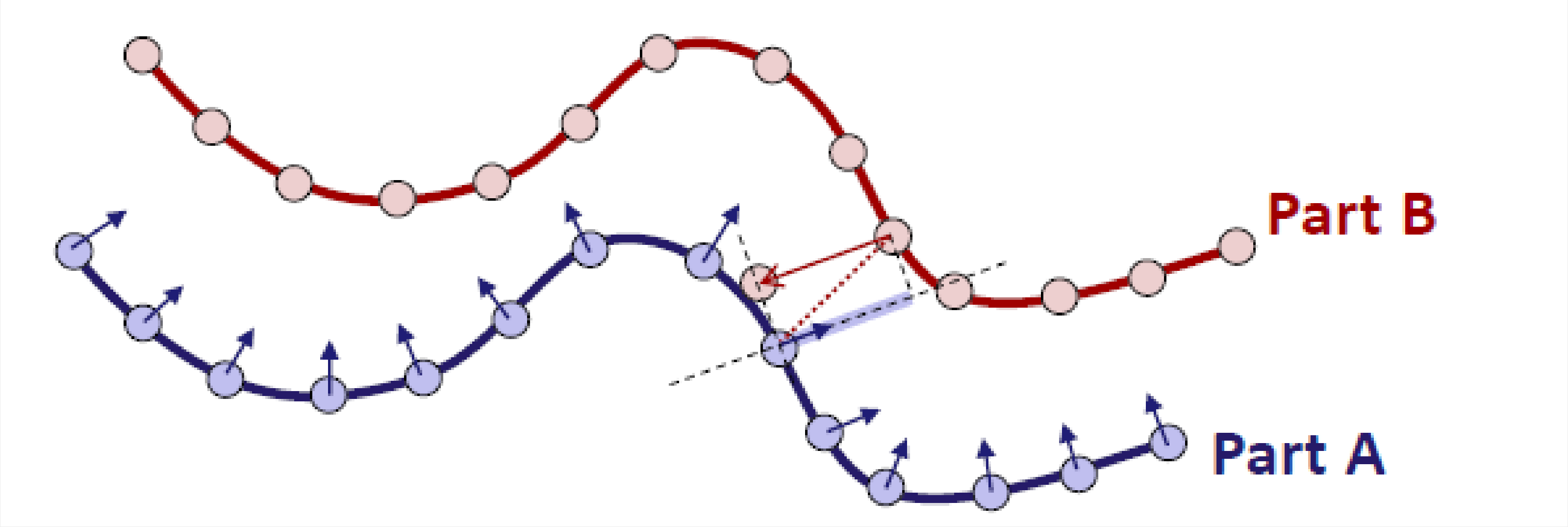


Generalizations (1)

- “point-to-plane” ICP
- First order approximation
  - Match points to tangential planes rather than points
  - Converges much faster



Generalizations (2)



Implementation:

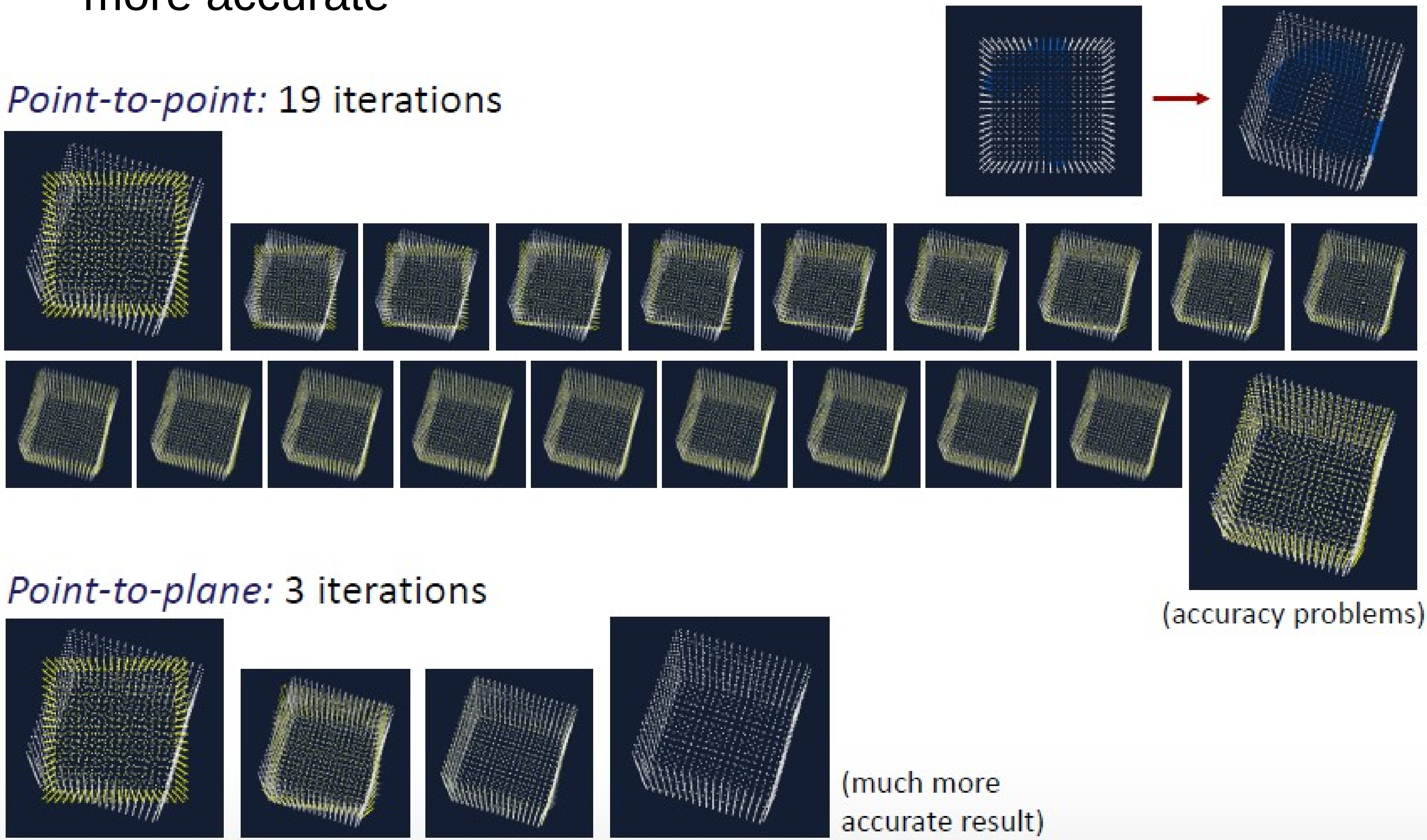
- We need normals for each point (unoriented/oriented)
- Compute closes point along normal direction

  or

- Compute closest point as usual, project it to surface defined by query point and normal
- Desirable: reduced points with normals

Comparisons

- In literature it is claimed, that point-to-plane is faster and more accurate



More Tricks and Tweaks

- ICP Problems:
  - Partial matching might lead to distortions / bias
  - Remove outliers
    - M-estimator
    - delete “far away points”, e.g. 20% percentile in point-to-point distance or
    - hard point-to-point distance threshold (for environments 20cm)
  - Remove normal outliers (if connection direction deviates from normal direction)
- Sampling problems
  - Problem: for example flat surface with engraved letters
  - No convergence in that case
  - Improvement: Sample correspondence points with distribution to cover unit sphere of normal directions as uniformly as possible

More Tricks and Tweaks

- ICP Problems:
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- Sampling problems
  - Problem: for example flat surface with engraved letters
  - No convergence in that case
  - Improvement: Sample correspondence points with distribution to cover unit sphere of normal directions as uniformly as possible

Things to try...

bin/slam6D dat

bin/show dat

bin/slam6D -r 10 dat

bin/show dat

Processing Large Data Sets (1)

```
bin/slam6D -s 1 -e 65 -r 10 -i 100 -d 75
        --epsICP=0.00001 ~/dat/hannover/
```

We see: small matching errors accumulate

6D SLAM – Global Relaxation (1)

- In SLAM loop closing is the key to build consistent maps
- Notice: Consistent vs. correct or accurate

- GraphSLAM
  - Graph Estimation
  - Graph Optimization

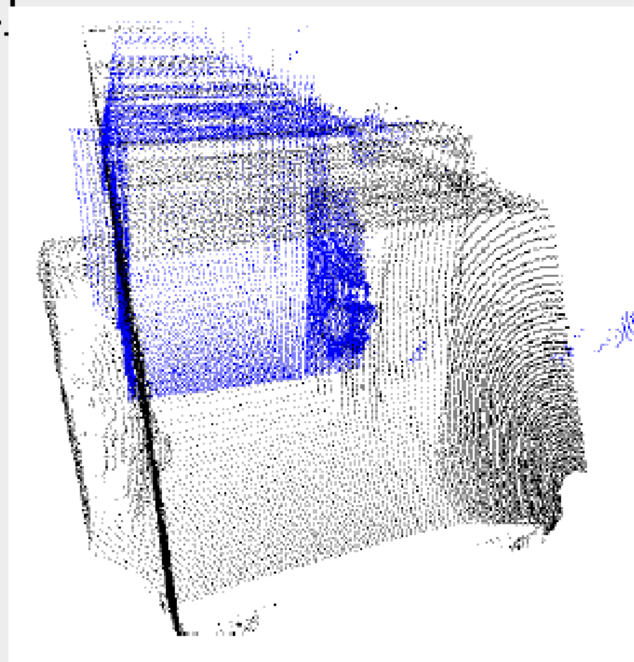
- Graph Estimation
  - Simple strategy: Connect poses with graph edges that are close enough
  - Simple strategy: Connect poses, they have enough point pairs (closest points)

The ICP Algorithm

The global ICP Algorithm

Scan registration Put two independent scans into one frame of reference

Iterative Closest Point algorithm [Besl/McKay 1992]



For prior point set *M* (“model set”) and data set *D*

1. Select point correspondences *wi,j* in {0,1}
2. Minimize for rotation **R**, translation **t**

$$E(\mathbf{R},\mathbf{t}) = \sum_{i=1}^{N_m}\sum_{j=1}^{N_d} w_{i,j}\,||\mathbf{m}_i - (\mathbf{R}\mathbf{d}_j + \mathbf{t})||^2$$

3. Iterate **1.** and **2.**

Four closed form solution for the minimization works in 3 translation plus 3 rotation dimensions

For globally consistent scan matching use the following error function

$$E = \sum_{j \rightarrow k}\sum_{i} \left|\mathbf{R}_j\mathbf{m}_i + \mathbf{t}_j - (\mathbf{R}_k\mathbf{d}_i + \mathbf{t}_k)\right|^2$$

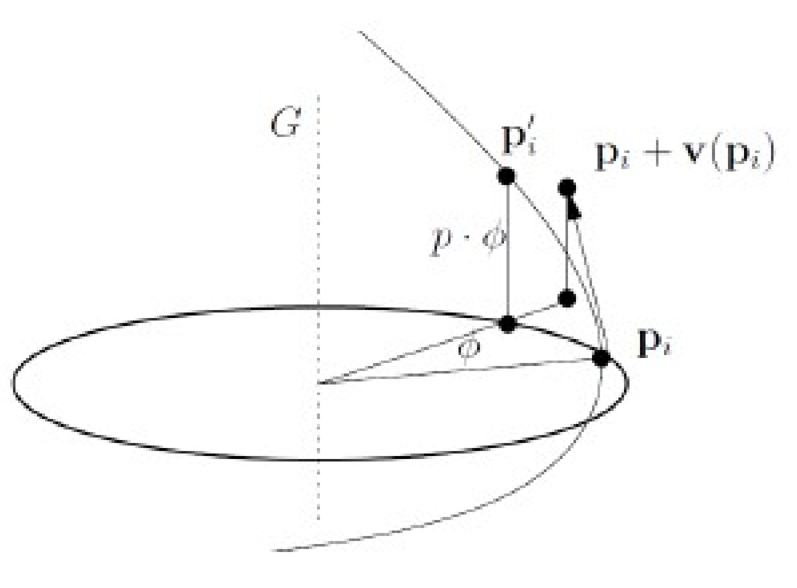
Minimize for all rotations **R** and translations **t** at the same time

Parametrizations for the Rigid Body Transformations

$$E = \sum_{j \to k} \sum_{i} \left| \mathbf{R}_j \mathbf{m}_i + \mathbf{t}_j - (\mathbf{R}_k \mathbf{d}_i + \mathbf{t}_k) \right|^2$$

- Helix transformation

$$\mathbf{v}(\mathbf{p}) = \bar{\mathbf{x}} + \mathbf{x} \times \mathbf{p}$$



$$E = \sum_{j \to k} \sum_{i} (\mathbf{m}_i - \mathbf{d}_i + (\bar{\mathbf{x}}_j + \mathbf{x}_j \times \mathbf{m}_i) - (\bar{\mathbf{x}}_k + \mathbf{x}_k \times \mathbf{m}_i))^2$$

… solving a system of linear equations

Parametrizations for the Rigid Body Transformations

$$E = \sum_{j \to k} \sum_i |\mathbf{R}_j \mathbf{m}_i + \mathbf{t}_j - (\mathbf{R}_k \mathbf{d}_i + \mathbf{t}_k)|^2$$

- Small angle approximation

$$\sin\theta \approx \theta - \frac{\theta^3}{3} + \frac{\theta^5}{5} - \cdots$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2} + \frac{\theta^4}{4} - \cdots$$

$$\mathbf{R} \approx \begin{pmatrix} 1 & -\theta_z & \theta_y \\ \theta_x\theta_y + \theta_z & 1 - \theta_x\theta_y\theta_z & -\theta_x \\ \theta_x\theta_z - \theta_y & \theta_x + \theta_y\theta_z & 1 \end{pmatrix}$$

$$\mathbf{R} \approx \begin{pmatrix} 1 & -\theta_z & \theta_y \\ \theta_z & 1 & -\theta_x \\ -\theta_y & \theta_x & 1 \end{pmatrix}$$

... solving a system of linear equations

Parametrizations for the Rigid Body Transformations

$$E = \sum_{j \to k} \sum_i \left| \mathbf{R}_j \mathbf{m}_i + \mathbf{t}_j - (\mathbf{R}_k \mathbf{d}_i + \mathbf{t}_k) \right|^2$$

- Explicit modeling of uncertainties
- Assumptions: The unknown error is normally distributed

$$\begin{aligned} W &= \sum_{j \to k} (\bar{\mathbf{E}}_{j,k} - \mathbf{E}'_{j,k})^T \mathbf{C}_{j,k}^{-1} (\bar{\mathbf{E}}'_{j,k} - \mathbf{E}'_{j,k}) \\ &= \sum_{j \to k} (\bar{\mathbf{E}}_{j,k} - (\mathbf{X}'_j - \mathbf{X}'_k)) \mathbf{C}_{j,k}^{-1} (\bar{\mathbf{E}}'_{j,k} - (\mathbf{X}'_j - \mathbf{X}'_k)). \end{aligned}$$

$$E_{j,k} = \sum_{i=1}^{m} \| \mathbf{X}_j \oplus \mathbf{d}_i - \mathbf{X}_k \oplus \mathbf{m}_i \|^2 = \sum_{i=1}^{m} \| \mathbf{Z}_i(\mathbf{X}_j, \mathbf{X}_k) \|^2$$

… solving a system of linear equations

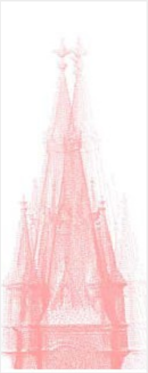
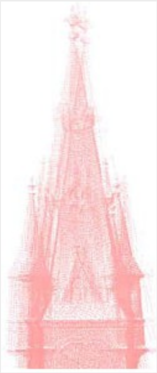
Comparisons of the Parametrizations

Global ICP	Classical Pose GraphSLAM
• Gaussian noise in the „3D Point Cloud“ space	• Gaussian noise in the space of poses
• Locally optimal	• Gradient descent needed
• ICP-like iterations using new point correspondences	• ICP-like iterations using new point correspondences needed as well

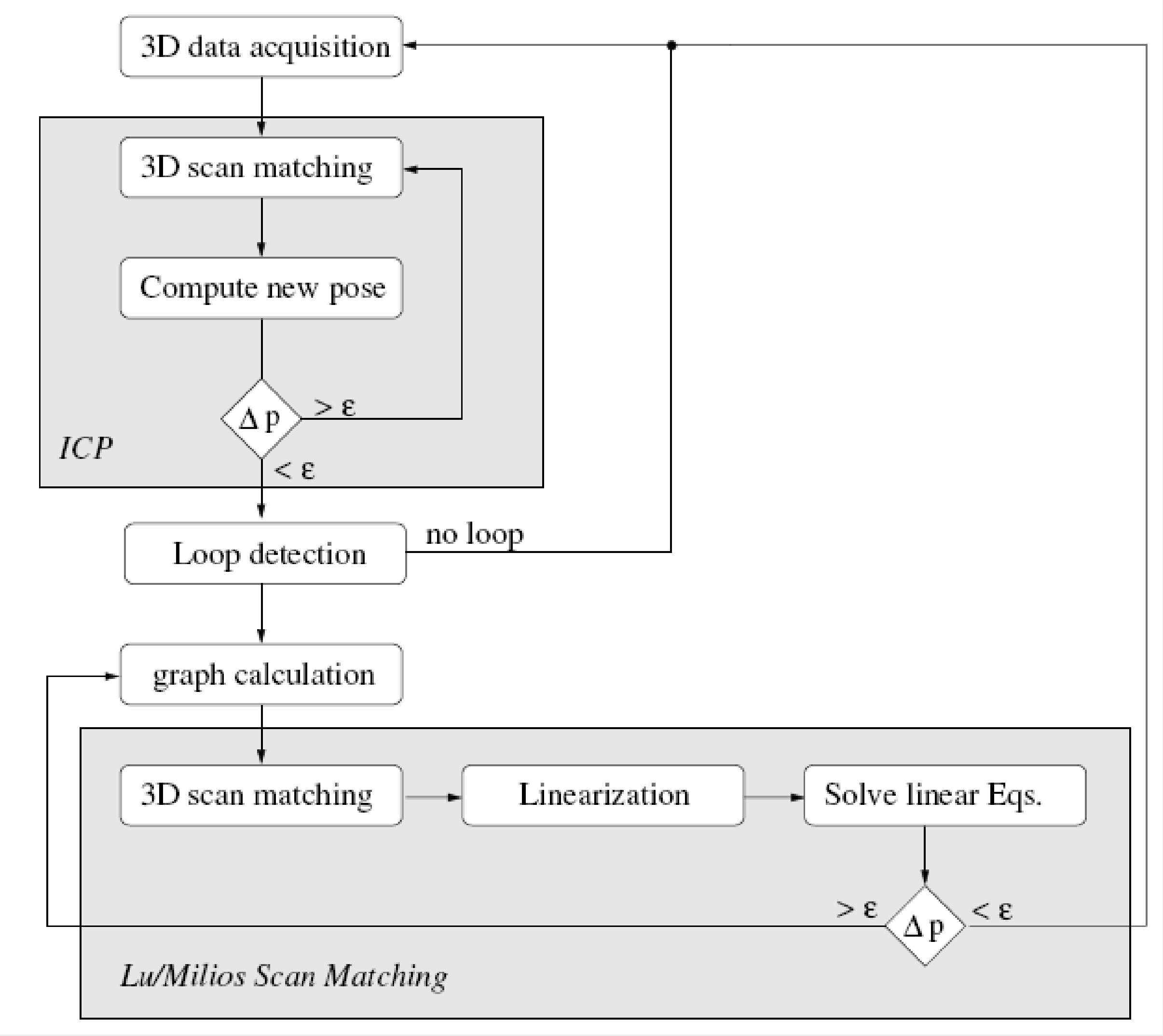
- Riegl Laser Measurement GmbH

(video)

(video) (video) (video)

Closed Loop Detection and Global Relaxation



Processing Large Data Sets (2)

```
bin/slam6D -s 1 -e 65 -r 10 -i 100 -d 75
        --epsICP=0.00001 ~/dat/hannover/
```

We see: small matching errors accumulate

```
bin/slam6D -s 1 -e 65 -r 10 -i 100 -d 75
        --epsICP=0.00001
        -D 250 -I 50 --cldist=750 -L 0 -G 1            ~/dat_hannover

bin/show -s 1 -e 65 ~/dat/dat_hannover
```